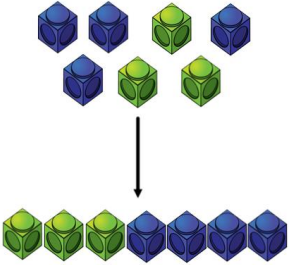
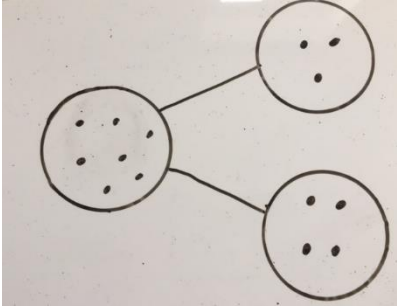
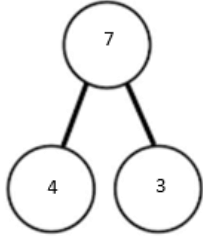
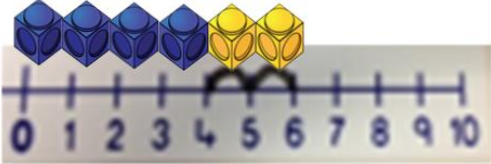
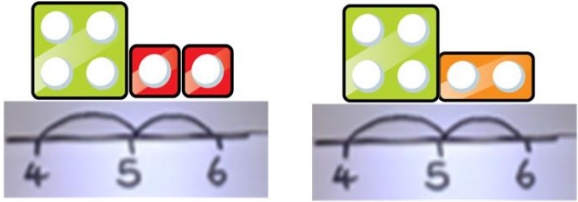
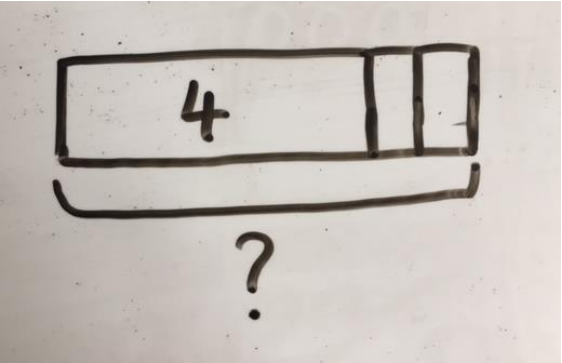



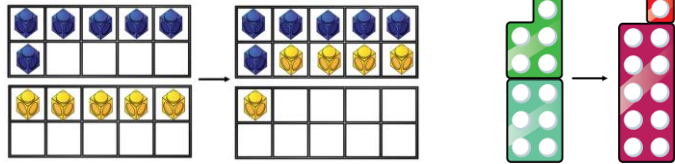
# Calculation policy: Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

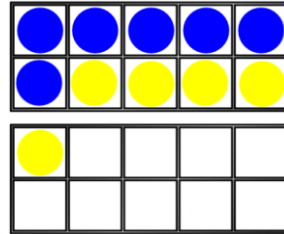
Concrete	Pictorial	Abstract
<p><b>Combining two parts to make a whole</b> (use other resources too e.g. eggs, shells, teddy bears, cars).</p> 	<p>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p> 	<p><math>4 + 3 = 7</math> Four is a part, 3 is a part and the whole is seven.</p> 
<p><b>Counting on using number lines</b> using cubes or Numicon.</p>  	<p>A bar model which encourages the children to count on, rather than count all.</p> 	<p>The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? <math>4 + 2</math></p> 

**Regrouping to make 10;** using ten frames and counters/cubes or using Numicon.

6 + 5



Children to draw the ten frame and counters/cubes.



Children to develop an understanding of equality e.g.

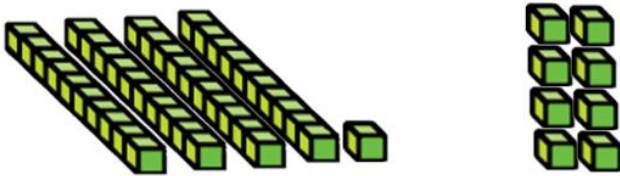
$$6 + \square = 11$$

$$6 + 5 = 5 + \square$$

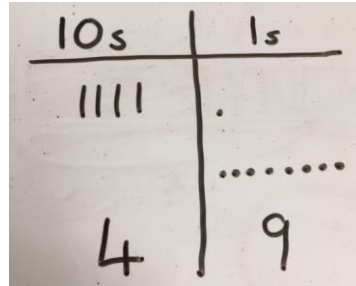
$$6 + 5 = \square + 4$$

**TO + O using base 10.** Continue to develop understanding of partitioning and place value.

41 + 8



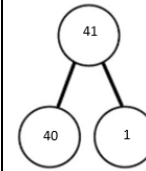
Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.



41 + 8

$$1 + 8 = 9$$

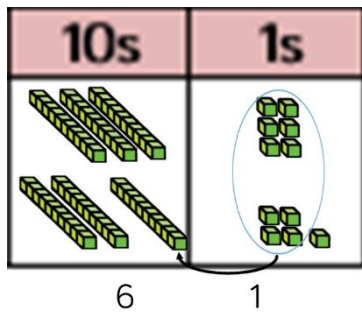
$$40 + 9 = 49$$



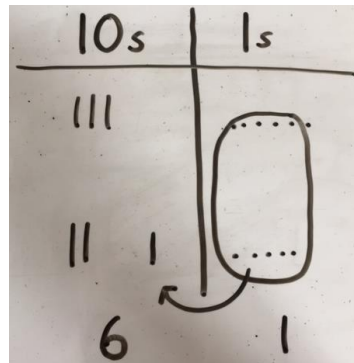
	4	1
+		8
<hr/>		
	4	9

**TO + TO using base 10.** Continue to develop understanding of partitioning and place value.

36 + 25



Children to represent the base 10 in a place value chart.



Looking for ways to make 10.

36 + 25 =

$$30 + 20 = 50$$

$$5 + 5 = 10$$

$$50 + 10 + 1 = 61$$

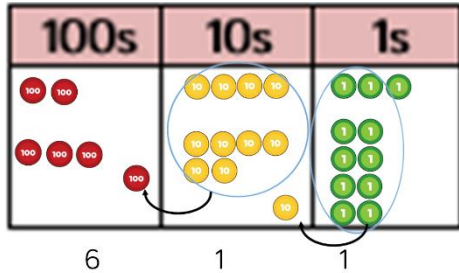
1      5

36

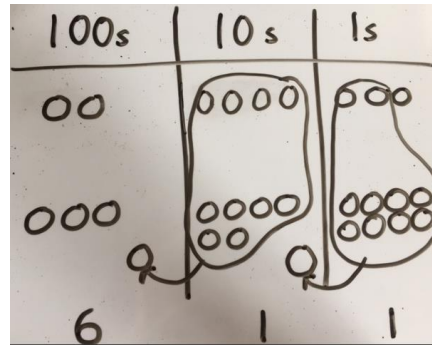
Formal method:

	<u>+25</u>
	61
	<hr/>
	1

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.

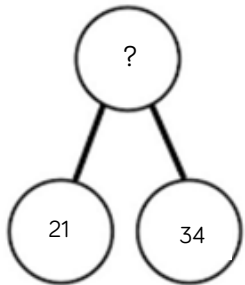


Children to represent the counters in a place value chart, circling when they make an exchange.



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 11 \end{array}$$

## Conceptual variation; different ways to ask children to solve $21 + 34$



?	
21	34

Word problems:

In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

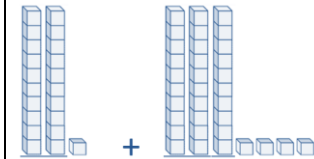
$21 + 34 = 55$ . Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$21 + 34 =$

$$\square = 21 + 34$$

Calculate the sum of twenty-one and thirty-four.

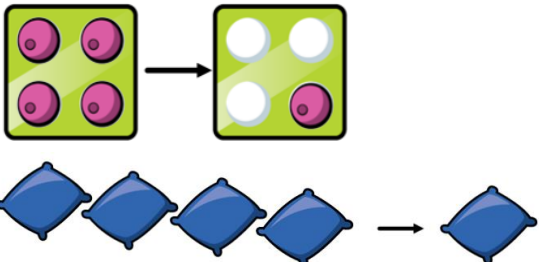
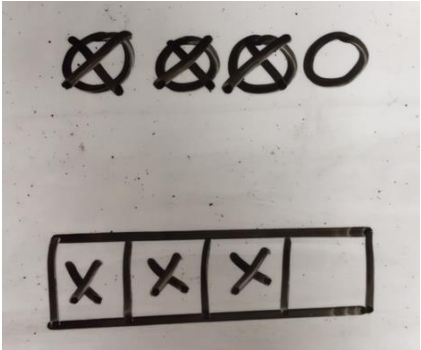

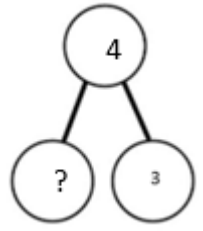
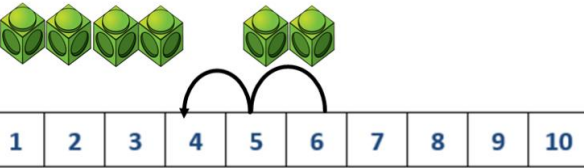
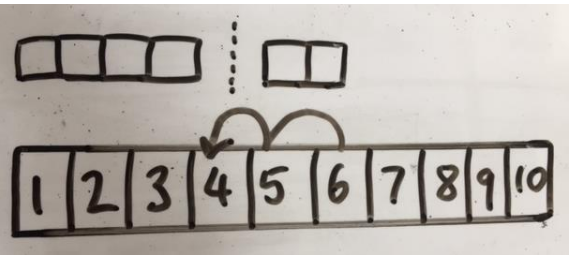


Missing digit problems:

10s	1s
10 10	1
10 10 10	?
?	5

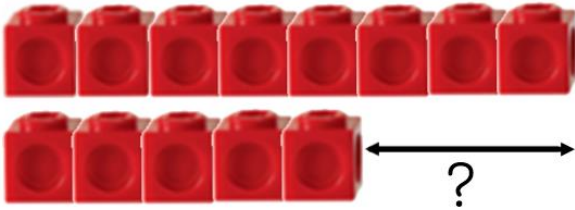
# Calculation policy: Subtraction

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

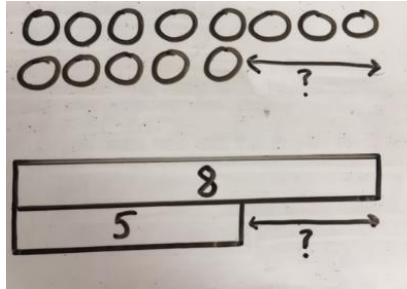
Concrete	Pictorial	Abstract
<p><b>Physically taking away and removing objects from a whole</b> (ten frames, Numicon, cubes and other items such as beanbags could be used).</p> <p><math>4 - 3 = 1</math></p>  <p>The image shows two stages of a subtraction activity. In the first stage, a green ten frame contains four pink circles, and four blue beanbags are lined up below it. An arrow points to the second stage, where the ten frame now contains one pink circle and three white circles, and only one blue beanbag remains.</p>	<p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p>  <p>The image shows a hand-drawn pictorial representation. At the top, four circles are drawn, with the first three crossed out with an 'X'. Below this, a horizontal bar is divided into four equal sections, with the first three sections also marked with an 'X'.</p>	<p><math>4 - 3 =</math></p>  <p>The image shows an abstract representation. At the top, the equation <math>4 - 3 =</math> is written next to a dashed square box. Below this is a bar model with a top section labeled '4', a bottom-left section labeled '3', and an empty bottom-right section labeled '?'. Below the bar model is a number bond with a top circle labeled '4', a bottom-left circle labeled '?', and a bottom-right circle labeled '3'.</p>
<p><b>Counting back</b> (using number lines or number tracks) children start with 6 and count back 2.</p> <p><math>6 - 2 = 4</math></p>  <p>The image shows a concrete subtraction activity. At the top, six green cubes are arranged in two groups of three. Below them is a number line from 1 to 10. Two curved arrows are drawn above the number line, starting at 6 and jumping back to 5, and then from 5 to 4.</p>	<p>Children to represent what they see pictorially e.g.</p>  <p>The image shows a hand-drawn pictorial representation. At the top, a number line from 1 to 10 is drawn. Above the number line, there are two groups of boxes: one group of three boxes and one group of two boxes. A vertical dashed line is drawn between the two groups. Below the number line, two curved arrows are drawn, starting at 6 and jumping back to 5, and then from 5 to 4.</p>	<p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line</p>  <p>The image shows two examples of abstract representations. The top one is a number line from 0 to 10 with a curved arrow starting at 6 and jumping back to 4. The bottom one is a number line with a curved arrow starting at 6 and jumping back to 4.</p>

**Finding the difference** (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5.



Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.



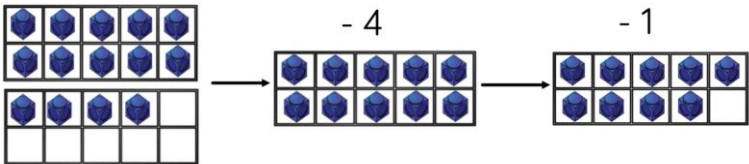
Find the difference between 8 and 5.

8 - 5, the difference is

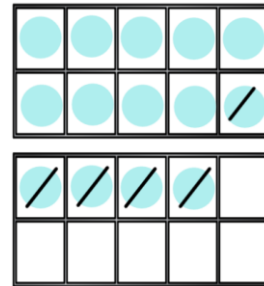
Children to explore why  
 $9 - 6 = 8 - 5 = 7 - 4$  have the same difference.

**Making 10** using ten frames.

$14 - 5$



Children to present the ten frame pictorially and discuss what they did to make 10.



Children to show how they can make 10 by partitioning the subtrahend.

$$14 - 5 = 9$$

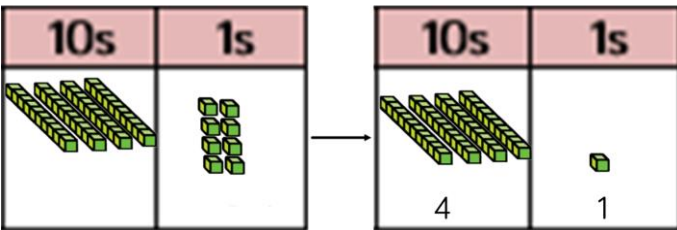
$$\begin{array}{c} 4 \quad 1 \end{array}$$

$$14 - 4 = 10$$

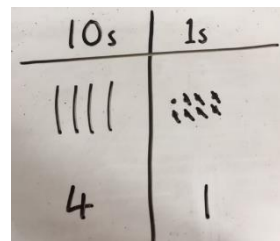
$$10 - 1 = 9$$

**Column method** using base 10.

$48 - 7$



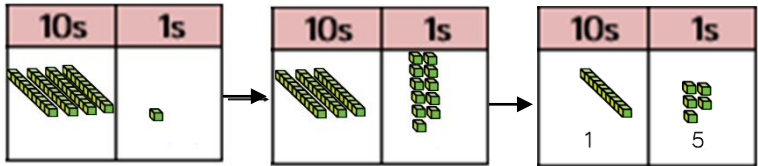
Children to represent the base 10 pictorially.



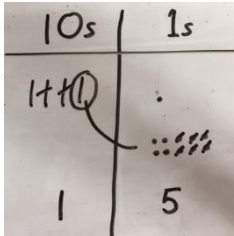
Column method or children could count back 7.

	4	8
-		7
	4	1

Column method using base 10 and having to exchange.  
41 - 26



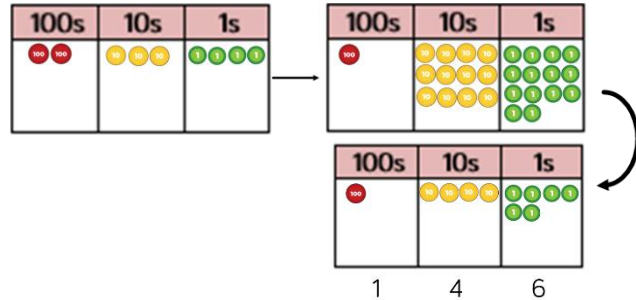
Represent the base 10 pictorially, remembering to show the exchange.



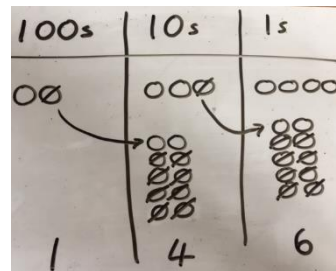
Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because  $41 = 30 + 11$ .

$$\begin{array}{r} 3 \cancel{4} 1 \\ - 26 \\ \hline 15 \end{array}$$

Column method using place value counters.  
234 - 88



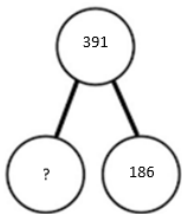
Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal column method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} 2 \cancel{3} 4 \\ - 88 \\ \hline 6 \end{array}$$

## Conceptual variation; different ways to ask children to solve $391 - 186$



391	
186	?

Raj spent £391, Timmy spent £186.  
How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\square = 391 - 186$$

$$\begin{array}{r} 391 \\ - 186 \\ \hline \end{array}$$

What is 186 less than 391?

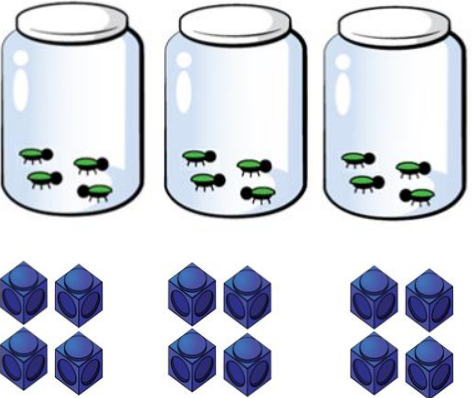
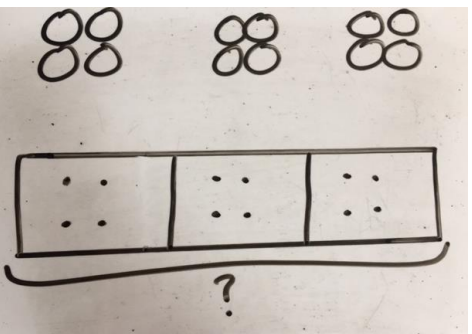
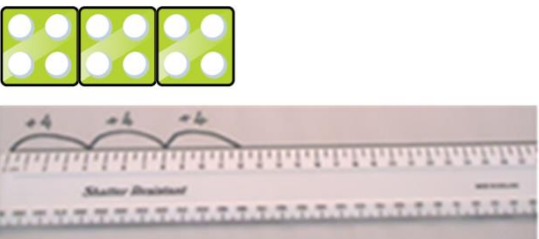
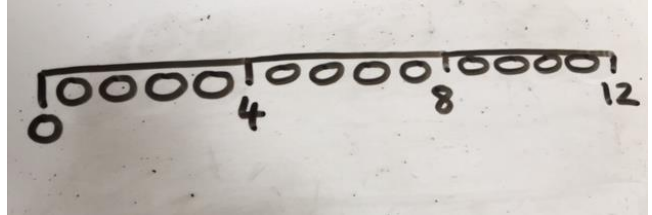
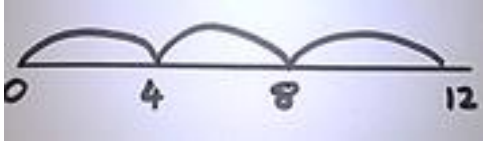
Missing digit calculations

$$\begin{array}{r} 39\square \\ - \square\square 6 \\ \hline \square 05 \end{array}$$



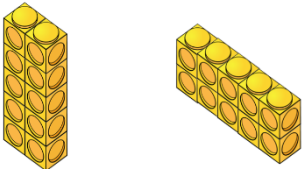
# Calculation policy: Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Concrete	Pictorial	Abstract
<p><b>Repeated grouping/repeated addition</b> <math>3 \times 4</math> <math>4 + 4 + 4</math> There are 3 equal groups, with 4 in each group.</p>  <p>The concrete representation shows three identical jars, each containing four small green beetles. Below the jars are three groups of four blue cubes, arranged in two rows of two cubes per group.</p>	<p>Children to represent the practical resources in a picture and use a bar model.</p>  <p>The pictorial representation shows three groups of two circles drawn together. Below this is a hand-drawn bar model divided into three equal sections, each containing two dots. A bracket underneath the bar model is labeled with a question mark.</p>	<p><math>3 \times 4 = 12</math> <math>4 + 4 + 4 = 12</math></p>
<p><b>Number lines to show repeated groups-</b> <math>3 \times 4</math></p>  <p>The concrete representation shows three green Cuisenaire rods, each representing a group of four units. Below them is a ruler with three jumps of four units each, starting from 0 and ending at 12.</p> <p>Cuisenaire rods can be used too.</p>	<p>Represent this pictorially alongside a number line e.g.:</p>  <p>The pictorial representation shows a hand-drawn number line from 0 to 12. There are three jumps of four units each, starting from 0 and ending at 12. The numbers 4, 8, and 12 are marked on the line.</p>	<p>Abstract number line showing three jumps of four.</p> <p><math>3 \times 4 = 12</math></p>  <p>The abstract number line shows a horizontal line from 0 to 12. There are three jumps of four units each, starting from 0 and ending at 12. The numbers 4, 8, and 12 are marked on the line.</p>

Use arrays to illustrate commutativity counters and other objects can also be used.

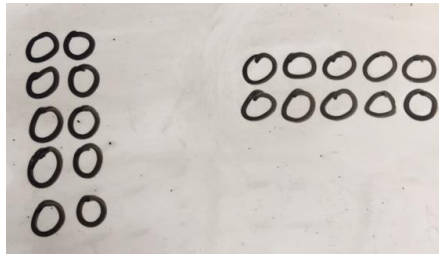
$$2 \times 5 = 5 \times 2$$



2 lots of 5

5 lots of 2

Children to represent the arrays pictorially.



Children to be able to use an array to write a range of calculations e.g.

$$10 = 2 \times 5$$

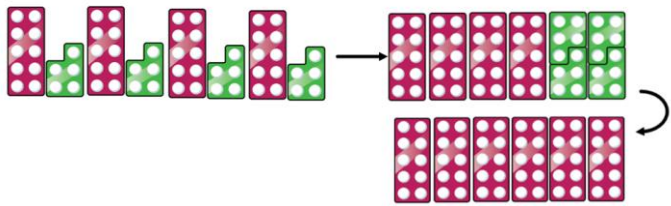
$$5 \times 2 = 10$$

$$2 + 2 + 2 + 2 + 2 = 10$$

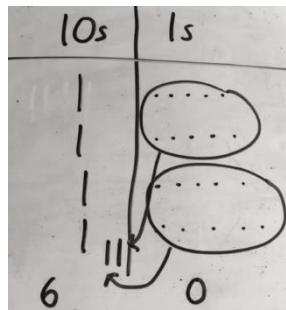
$$10 = 5 + 5$$

Partition to multiply using Numicon, base 10 or Cuisenaire rods.

$$4 \times 15$$



Children to represent the concrete manipulatives pictorially.



Children to be encouraged to show the steps they have taken.

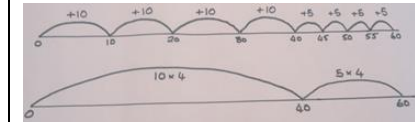
$$4 \times 15$$

$$10 \times 4 = 40$$

$$5 \times 4 = 20$$

$$40 + 20 = 60$$

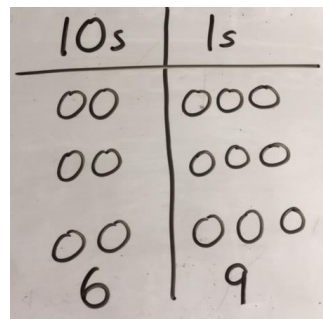
A number line can also be used



Formal column method with place value counters (base 10 can also be used.)  $3 \times 23$

10s	1s
6	9

Children to represent the counters pictorially.



Children to record what it is they are doing to show understanding.

$$3 \times 23$$

$$3 \times 20 = 60$$

$$3 \times 3 = 9$$

$$20 \quad 3$$

$$60 + 9 = 69$$

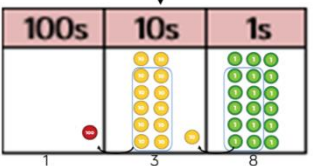
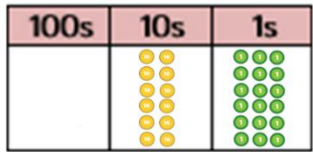
23

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

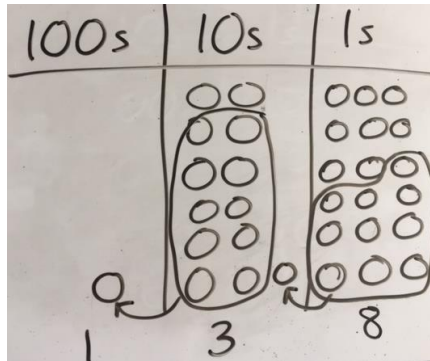


Formal column method with place value counters.

$6 \times 23$



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method

$$\begin{array}{r}
 6 \times 23 = \\
 23 \\
 \times 6 \\
 \hline
 138 \\
 \hline
 11
 \end{array}$$

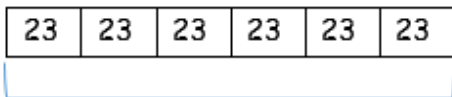
When children start to multiply  $3d \times 3d$  and  $4d \times 2d$  etc., they should be confident with the abstract:

To get 744 children have solved  $6 \times 124$ .  
 To get 2480 they have solved  $20 \times 124$ .

$$\begin{array}{r}
 124 \\
 \times 26 \\
 \hline
 744 \\
 2480 \\
 \hline
 3224 \\
 11
 \end{array}$$

Answer: 3224

## Conceptual variation; different ways to ask children to solve $6 \times 23$



?

Mai had to swim 23 lengths, 6 times a week.  
 How many lengths did she swim in one week?

With the counters, prove that  $6 \times 23 = 138$

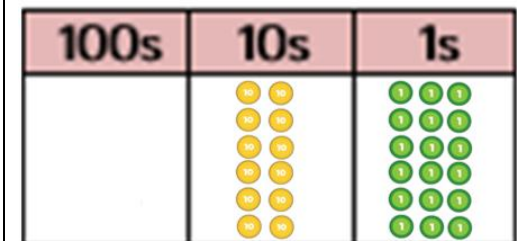
Find the product of 6 and 23

$6 \times 23 =$

$\square = 6 \times 23$

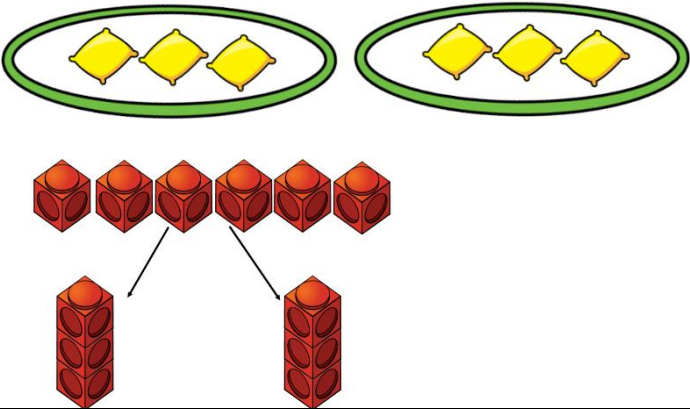
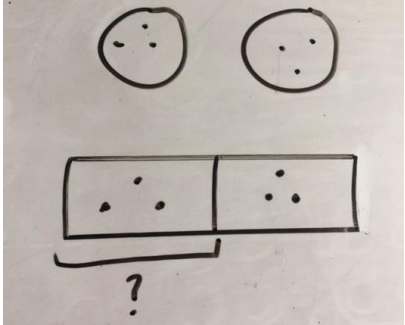
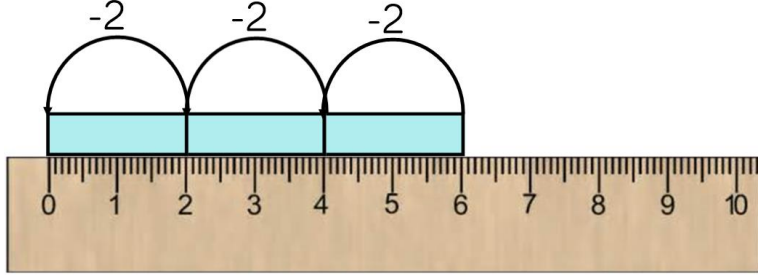
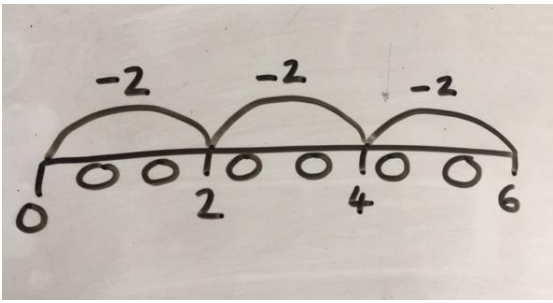
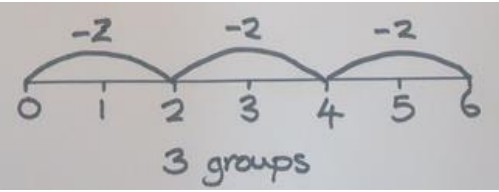
$$\begin{array}{r}
 6 \quad 23 \\
 \times \quad 23 \quad \times 6 \\
 \hline
 \quad \quad \quad \hline
 \end{array}$$

What is the calculation?  
 What is the product?



# Calculation policy: Division

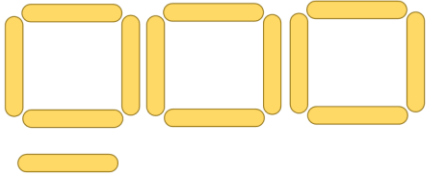
Key language: share, group, divide, divided by, half.

Concrete	Pictorial	Abstract		
<p><b>Sharing</b> using a range of objects. <math>6 \div 2</math></p>  <p>The diagram shows two green ovals, each containing three yellow diamonds. Below this, six red Cuisenaire rods are arranged in a row. Two arrows point from the first and fourth rods to two separate vertical stacks of three rods each, representing two groups of three.</p>	<p>Represent the sharing pictorially.</p>  <p>The diagram shows two hand-drawn circles, each containing three dots. Below them is a hand-drawn rectangle divided into two equal halves, each containing three dots. A bracket under the first half is labeled with a question mark, indicating the unknown number of groups.</p>	<p><math>6 \div 2 = 3</math></p> <table border="1" data-bbox="1554 480 2007 549"><tr><td>3</td><td>3</td></tr></table> <p>Children should also be encouraged to use their 2 times tables facts.</p>	3	3
3	3			
<p><b>Repeated subtraction</b> using Cuisenaire rods above a ruler. <math>6 \div 2</math></p>  <p>The diagram shows a ruler from 0 to 10. Three light blue Cuisenaire rods are placed above the ruler, each spanning from 0 to 2. Three arcs labeled '-2' are drawn above the rods, indicating the subtraction of 2 from 6. Below the ruler, the text '3 groups of 2' is written.</p>	<p>Children to represent repeated subtraction pictorially.</p>  <p>The diagram shows a horizontal line with six small circles below it. The circles are numbered 0, 2, 4, and 6. Three arcs labeled '-2' are drawn above the line, starting at 0 and ending at 2, 2 and 4, and 4 and 6, representing the repeated subtraction of 2 from 6.</p>	<p>Abstract number line to represent the equal groups that have been subtracted.</p>  <p>The diagram shows a number line from 0 to 6. Three arcs labeled '-2' are drawn above the line, starting at 0 and ending at 2, 2 and 4, and 4 and 6. Below the line, the text '3 groups' is written.</p>		

**2d ÷ 1d with remainders** using lollipop sticks. Cuisenaire rods, above a ruler can also be used.

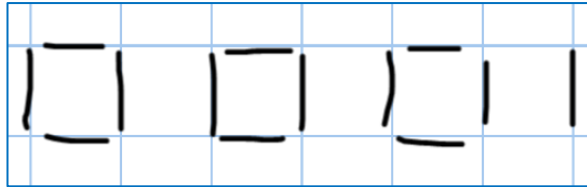
$$13 \div 4$$

Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.

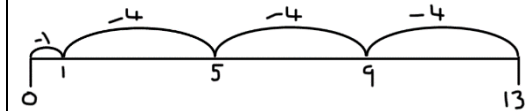


There are 3 whole squares, with 1 left over.

$$13 \div 4 = 3 \text{ remainder } 1$$

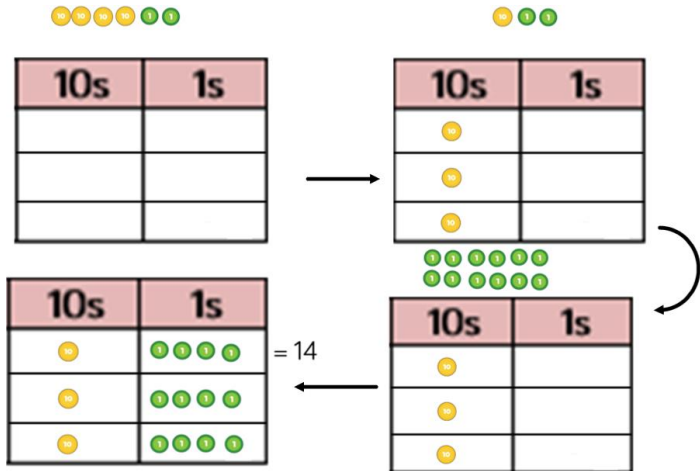
Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

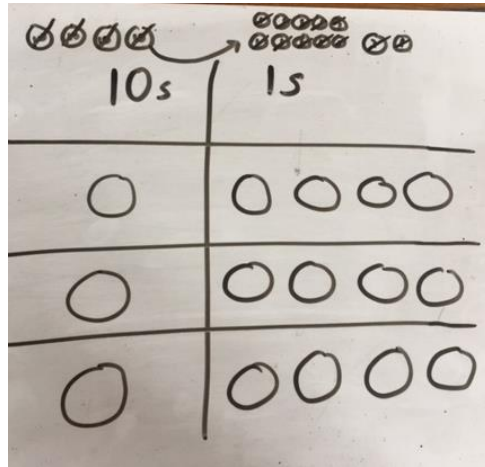


**Sharing using place value counters.**

$$42 \div 3 = 14$$



Children to represent the place value counters pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

$$42 \div 3$$

$$42 = 30 + 12$$

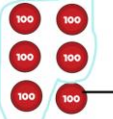
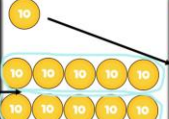

$$30 \div 3 = 10$$

$$12 \div 3 = 4$$

$$10 + 4 = 14$$

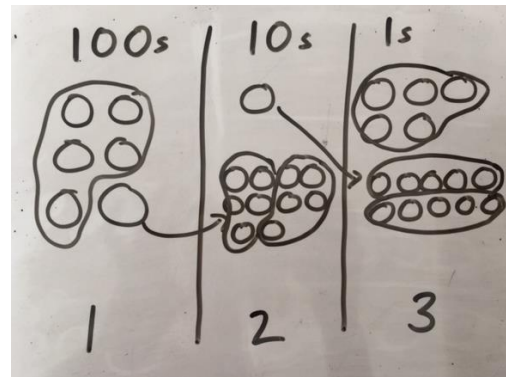
**Short division** using place value counters to group.

$$615 \div 5$$

100s	10s	1s
		
1	2	3

1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



Children to the calculation using the short division scaffold.




$$\begin{array}{r}
 123 \\
 5 \overline{) 615} \\
 \underline{6} \phantom{0} \\
 1 \phantom{0} \\
 \underline{10} \\
 15 \\
 \underline{15} \\
 0
 \end{array}$$

**Long division** using place value counters

$$2544 \div 12$$

1000s	100s	10s	1s
			

We can't group 2 thousands into groups of 12 so will exchange them.

1000s	100s	10s	1s
			

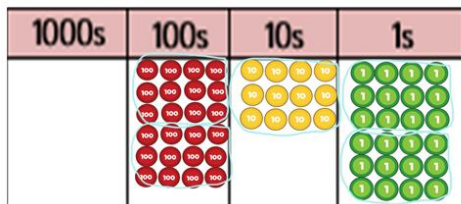
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r}
 02 \\
 12 \overline{) 2544} \\
 \underline{24} \phantom{0} \\
 1 \phantom{0}
 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r}
 021 \\
 12 \overline{) 2544} \\
 \underline{24} \phantom{0} \\
 14 \phantom{0} \\
 \underline{12} \phantom{0} \\
 2
 \end{array}$$

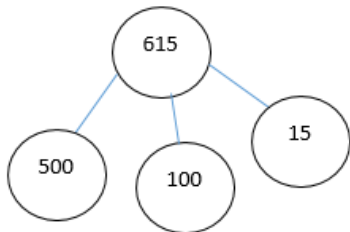


After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r}
 0212 \\
 12 \overline{) 2544} \\
 \underline{24} \phantom{0} \\
 14 \phantom{0} \\
 \underline{12} \phantom{0} \\
 24 \\
 \underline{24} \\
 0
 \end{array}$$

## Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{) 615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation?  
What is the answer?

